International Journal of Marketing and Technology
(ISSN: 2249-1058)

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Editorial Board

November 2011 IJMT Volume 1, Issue 6 ISSN: 2249-1058

Title

USING FUZZY COGNITIVE MAPS AND FUZZY RELATIONAL MAPS TO ANALYZE EMPLOYEE-EMPLOYER RELATIONSHIP IN AN INDUSTRY

Author(s)

Dhrubajyoti Ghosh

Departments of Mathematics,

National Institute of Technology Durgapur,

Durgapur-713209, West Bengal, India

Anita Pal

Departments of Mathematics,

National Institute of Technology Durgapur,

Durgapur-713209, West Bengal, India

Abstract:

The fuzzy model is a finite set of fuzzy relations that form an algorithm for determining the outputs of a process from some finite number of past inputs and outputs. Fuzzy model can be used in applied mathematics, to study social and psychological problem and also used by doctors, engineer, scientists, industrialists and statisticians. There are various types of fuzzy models. In this paper we use 2 fuzzy models and give their application to a real world problem. The paper is organized in four sections. 1st section defines cognitive maps. Section two defines Fuzzy Cognitive Maps (FCMs) and draws a relationship between employee and employer in an industry. In an industry, the employers expect to achieve performances in quality and production in order to earn profit, on the other side employees need good pay and all possible allowances and best advantages than any other industry. Section four gives the generalization and description of the FCMs; known as Fuzzy Relational Maps (FRMs) which describes a mathematical work of employee-employer relationship model which gives the maximizing production by giving maximum satisfaction to employee.

Keywords: Cognitive Maps (CMs), Fuzzy Cognitive Maps (FCMs), Employee-employer relationship model, Fuzzy Relational Maps (FRMs)

1. Cognitive Maps CMs):

Before the discussion of FCMs we describe about the CMs. CMs were introduced by Axelord (1976), in order to develop and study social scientific knowledge in the field of decision-making in activities related to international politics. CMs are signed digraphs designed to represent causal assertion and belief system of a person (or group of experts) with respect to a specific domain, and use that statement in order to analyze the effect of a certain choice on particular objectives.

2. Fuzzy Cognitive Maps (FCMs):

FCMs introduced by Kosko (1986) extend the idea of Cognitive Maps by allowing the concepts to be represented linguistically with an associated fuzzy set. FCMs are fuzzy signed digraph with feedback (Kosko, 1986, 1988). It represents causal relationship between concepts. FCMs link causal events, actor, value and trends to a fuzzy feedback dynamical system. FCMs list the fuzzy rule or causal flow paths that relate events.

3. Formation of FCMs:

If increase (or decrease) in one concept/node leads to increase (or decrease) in another concept, we give the value 1 and the vice versa, we give the value -1. If there exists no relation between concepts the value 0 is given. Consider the $C_1, C_2, \ldots, C_n$ be the nodes of the FCM. Suppose the directed graph is drawn using edge weight $e_{ij} \in \{0, 1, -1\}$. The matrix E be defined by $E = (e_{ij})$ where e_{ij} is the weight of the directed edge C_iC_j. E is called the adjacency matrix of the FCM. All matrices associated with an FCM are always square matrices with diagonal entries as zero. Now the instantaneous state vector $A = (a_1, a_2, \ldots, a_n)$ where $a_i \in \{0, 1\}$, and it denotes the on-off position of the node at an instant. $a_i = 0$ if a_i is off and $a_i = 1$ if a_i is on for $i = 1, 2, \ldots, n$. Let $C_1C_2, C_2C_3, \ldots, C_iC_j$ be the nodes of the edge of the FCM ($i \neq j$) form a directed cycle and FCM is said to be cyclic if it possesses a directed cycle. Otherwise it is acyclic.

FCM with cycles is said to have a feedback. When there is a feedback in an FCM, i.e., when the causal relations flow through a cycle in a revolutionary way, the FCM is called a dynamical system. When C_i is switched on and if the causality flows through the edges of a cycle and if it again causes C_i, we say that dynamical system goes round and round where $i = 1, 2, \ldots, n$. The equilibrium state for this dynamical system is called the hidden pattern. If the equilibrium state of a dynamical system is a unique state vector, then is called fixed point. If the FCM settles down with a state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1$, then this equilibrium is called a limit cycle. Finite number of FCMs can be combined together to produce the joint effect of all FCMs. Let $E_1, E_2, \ldots, E_p$ be the adjacency matrices of all the FCMs with nodes i.e. Combined FCMs denotes be the relational matrix by $E = E_1 + \ldots + E_p$. Suppose $A = (a_1, \ldots, a_n)$ is a vector which is passed into a dynamical system E. Then $AE = (a'_1, \ldots, a'_n)$ after thresholding and updating the vector suppose we get $(b_1, \ldots, b_n)$ we denote that by $(a'_1, \ldots, a'_n) \hookrightarrow (b_1, \ldots, b_n)$. Thus the symbol '$\hookrightarrow$' means the resultant vector has been threshold and updated.

4. Application of FCMs:

FCMs have been used in many different ways; some of them are listed below.

a) HIV/AIDS affected migrant labourers socio-economic problem

b) Maximum utilization of a time period in a day of the traffic route

c) Study of political situation

d) Study of probing causes of child labour

e) Study of employee-employer relationship

4.1 Model illustrating FCMs:

The employee-employer relationship is an intricate one. In an industry, the employers expect to achieve performances in quality and production in order to earn profit, on the other side employees need good pay and all possible allowances and best advantages than any other industry. Here we have to draw employee-employer relationship model. With the help of this model we find the hidden pattern of this model. This is very important, that we have several nodes and several opinions to draw various model of this relationship. Several models can give us best picture and clear idea of this model. Here we have to consider only one model. We use the following 16 nodes /concepts of FCM:-

$X_1 \rightarrow$ Pay with allowances and bonus to the employee

$X_2 \rightarrow$ Only pay to the employee

$X_3 \rightarrow$ Pay with allowances (or bonus) to the employee

$X_4 \rightarrow$ Best performance

$X_5 \rightarrow$ Average performance

$X_6 \rightarrow$ Poor performance

$X_7 \rightarrow$ Employee works more number of hours

$X_8 \rightarrow$ Employee works less number of hours

$X_9 \rightarrow$ Maximum profit to the industry

$X_{10} \rightarrow$ Only profit to the industry

$X_{11} \rightarrow$ Neither profit nor loss to the industry

$X_{12} \rightarrow$ Loss to the industry

$X_{13} \rightarrow$ Heavy loss to the industry

$X_{14} \rightarrow$ Stop work or strike by the employees

$X_{15} \rightarrow$ Relation between employee and employer

$X_{16} \rightarrow$ Demand of the employee which are not fulfilled

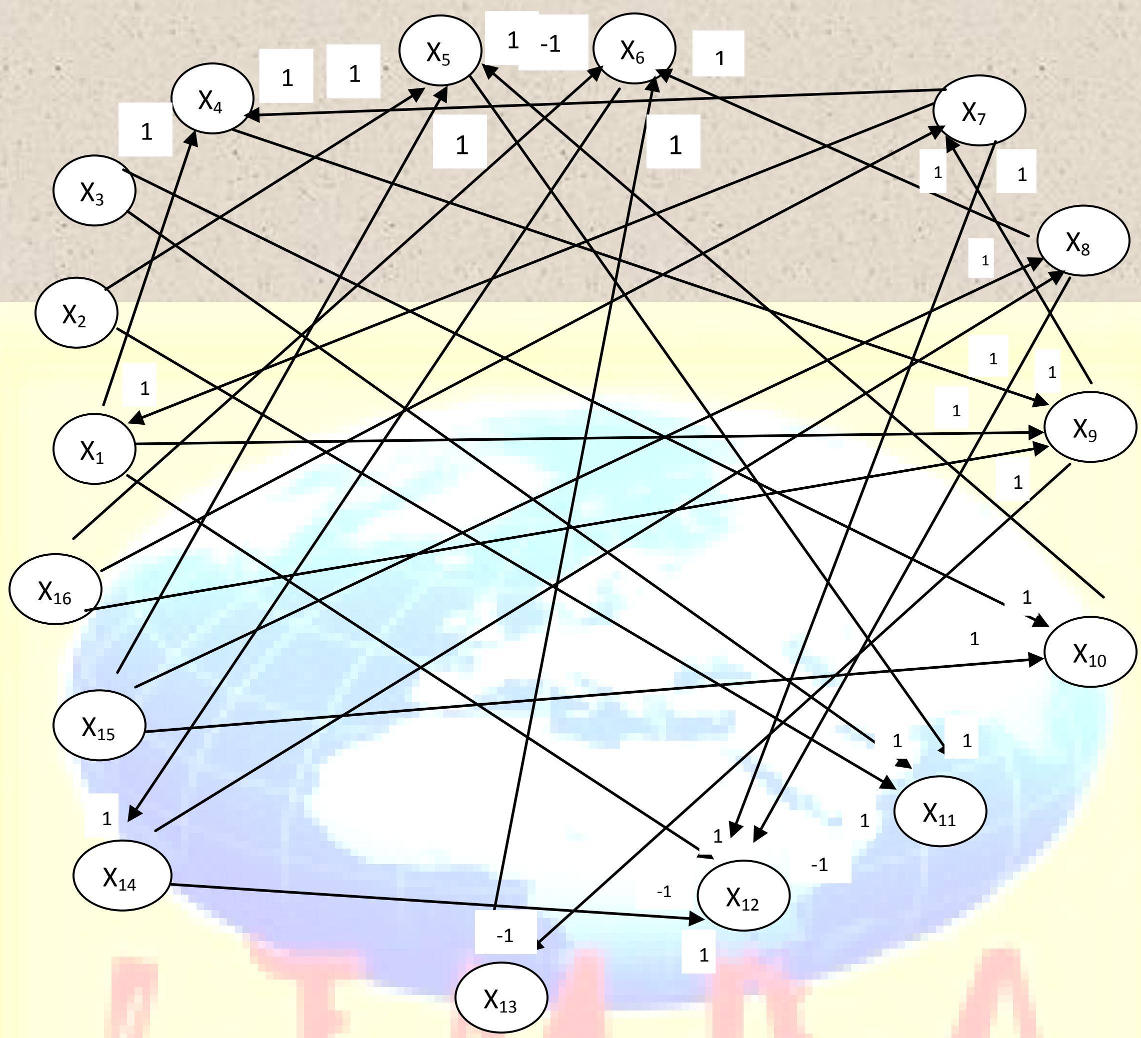

Fig 1: Employee -employer relationship model representation by a directed graph

With the help of this relational model we have to draw adjacency matrix.

X =

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	X_{16}
X_1	0	0	0	1	0	0	0	0	1	0	0	-1	0	0	0	0
X_2	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
X_3	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
X_4	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
X_5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
X_6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
X_7	1	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0
X_8	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
X_9	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0
X_{10}	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
X_{11}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{13}	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
X_{14}	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
X_{15}	0	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0
X_{16}	0	0	0	0	0	-1	1	0	1	0	0	0	0	0	0	0

TABLE 1: Adjacency matrix of FCMs drawn in Fig. 1

Here X denotes the connection matrix of the directed graph. Now we have to find the stability of the dynamical system or to be more precise the hidden pattern of the system may be a fixed point or a limit cycle. Considering one case on this relationship model with taking some nodes are on and some nodes are off and try to find out some hidden pattern.

CASE 1:

Let us take two nodes i.e." strike called by the employee (X_{14})" and "if the relation between employee and employer is not good or we have to say it is bad (X_{15})" to be the on state i.e. P_1 = (0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0) i.e. all other states are off state, passing P_1 to the connection matrix and of course after updating and thresholding the resultant vector we get,

P_1 = (0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0)

P_1X = (0 0 0 0 1 0 0 2 0 1 0 1 0 0 0 0)

(0 0 0 0 1 0 0 1 0 1 0 1 0 1 1 0) = P_2

P_2X = (0 0 0 0 2 1 0 2 0 1 0 2 0 0 0 0)

(0 0 0 0 1 1 0 1 0 1 0 1 0 1 1 0) = P_3

P_3X = (0 0 0 0 2 1 0 1 0 1 1 2 0 1 0 0)

(0 0 0 0 1 1 0 1 0 1 1 1 0 1 1 0) = P_4 = P_3

Here in node P_4 equilibrium state of dynamical system is a unique state vector Thus the result is fixed point. Now we have to say that "strike called by the employee (X_{14})" and "if the relation between employee and employer is not good (X_{15})" leads to "employee works less number of hours (X_8) ,so it will be affected the performance of an industry i.e. average performance (X_5) or may be it poor performance (X_6), so there will be affected the profit of the industry i.e. only profit to the industry (X_{10}), or neither profit nor loss to the industry (X_{11}), or may be loss to the industry (X_{13}). The others node are in off state.

Similarly we have to consider other cases from which we have to find the hidden pattern for the system.

4.2 <u>Conclusion:</u>

By this FCM model on employee-employer relationship in an industry and analysis the cases it is concluded that, if allowances and bonus are given to the employee, the employee will works more number of hours which maximizes the profit of the industry. But when the relation between employee and employer are not good and if the demands of the employee are not satisfied, the employee works in company with dissatisfaction, sometimes they may also call strikes which will hamper production, good will of the company and employee works less numbers of hours, so company will be running in loss. Good relation between employee and employer can change the atmosphere of the industry.

Now we have to discuss another type of fuzzy model

5. Definition of FRMs:

FRMs are a directed graph or a map from domain space to range space with concepts and causalities as edges.

Let, Domain space = n

Range space = m [$m \neq n$]

$R_1, R_2, \ldots, R_m$ be the nodes of range space.

$R = \{ (x_1, x_2, \ldots, x_m) \mid x_j = 0 \text{ or } 1\}$ for $j = 1,2\ldots,m$.

$x_j = 1$ i.e. R_j is on state

$x_j = 0$ i.e. R_j is off state

Similarly,

$D_1, D_2, \ldots, D_n$ be the nodes of domain space.

$D = \{ (x_1, x_2, \ldots, x_n) \mid x_i = 0 \text{ or } 1\}$ for $i = 1,2\ldots,n$.

$x_i = 1$ i.e. D_i is on state

$x_i = 0$ i.e. D_i is off state

5.1 Formation of FRMs:

Let D_i and R_j denotes the two nodes of FRM. Let e_{ij} be the weight of the edge D_iR_j (or R_jD_i), then $e_{ij} \in \{0, 1, -1\}$ The relational matrix E be defined as $E = (e_{ij})$.

a) Instantaneous state vector

Let $A = (a_1 \ldots, a_n)$, $a_i \in \{0, 1\}$ where $i = 1,2,..,n$. A is called the instantaneous state vector of the domain space and it denotes the on-off position of the nodes at any instant i.e. $a_i = 0$ if a_i is off and $a_i = 1$ if a_i is on for $i = 1,2,..,n$ for domain space.

Similarly, $B = (b_1, \ldots, b_m)$, $b_j \in \{0,1\}$ where $j = 1,2,..,m$. B is the instantaneous state vector of the range space. $b_j = 0$ if b_j is off and $b_j = 1$ if b_j is on for $j = 1,2,..,m$ for range space.

b) Directed Cycle

Let D_iR_j (or $R_j D_i$) be the edges of an FRM where $j = 1,2,\ldots,m$ and $i = 1,2,..,n$ form a directed cycle, FRM is said to be cycle if it possesses a directed cycle. Otherwise, it is acyclic.

c) Dynamical system of FRM

An FRM with cycle is said to be an FRM with feedback. When there is feedback in the FRM. The FRM is called a dynamical system.

d) Hidden pattern

Let D_iR_j (or R_j D_i) where $1 \leq j \leq m$ and $1 \leq i \leq n$. When D_j (or R_i) is switched on and if causality flows and if again causes D_j (or R_i). This equilibrium state is called hidden pattern.

e) Fixed Point

If the equilibrium state of this system is a unique state vector then it is called fixed point. Example: -Let us assume dynamical system by switching on R_1 (or D_1). FRM settles down with R_1 and R_m (D_1 and D_n) on i.e. state vector remains as (1,0,…0,1) as in R (1,0,..,0,1). This state vector is called the fixed point.

f) Limit cycle

If the FRM settles down with a state vector in the forms: $D_1 \to D_2 \to \ldots \to D_i \to D_1$ ($R_1 \to R_2 \to \ldots \to R_i \to R_1$). This form is called limit cycle.

g) Combined FRMs

Let us E_1, E_2,……, E_p be the relational matrices of the FRMs. Combined FRMs denotes be the relational matrix by $E = E_1 + \ldots + E_p$.

5.2 Methods of determining the hidden pattern

Let $R_1, R_2,\ldots,R_m$ and $D_1, D_2, \ldots, D_n$ be the nodes of FRM. Let us assume D_1 is switched on i.e. when an input is given as vector $A_1 = (1, 0,\ldots, 0)$ in $D_1$and the relational matrix is E. Now $A_1 E = (r_1, r_2,\ldots, r_m)$, after thresholding and updating the resultant vector $A_1E \in R$. Now let $B = A_1E$, passing B into E^T and obtain BE^T. After threshold and update the vector $BE^T \in D$. The procedure repeated till we get a fixed point or limit cycle.

5.3 Application of FRMs

FRMs are used in the following areas:-

a) Relation between Doctor and Patient.

b) Relation between quality condition and academic condition of students

c) Relational between teacher and poor rural students in City Colleges

d) Study of employee-employer relationship

5.4 Employee-employer relationship model developed by FRMs

At first, we have taken 8 nodes in domain space which denote the employee.

A_1 -> Pay with allowances and bonus to the employee

A_2 -> Only pay to the employee

A_3 -> Pay with allowances (or bonus) to the employee

A_4 -> Best performances by the employee

A_5-> Average performances by the employee

A_6-> Poor performances by the employee

A_7 -> Employee works for more number of hours

A_8-> Employee works for less number of hours

Now we have taken as 5 nodes related to the employer spaces which denote the range spaces.

B_1 -> Maximum profit to the employer

B_2 -> Only profit to the employer

B_3 -> Neither profit nor loss the employer

B_4 -> Loss to the employer

B_5 -> Heavy loss to the employee

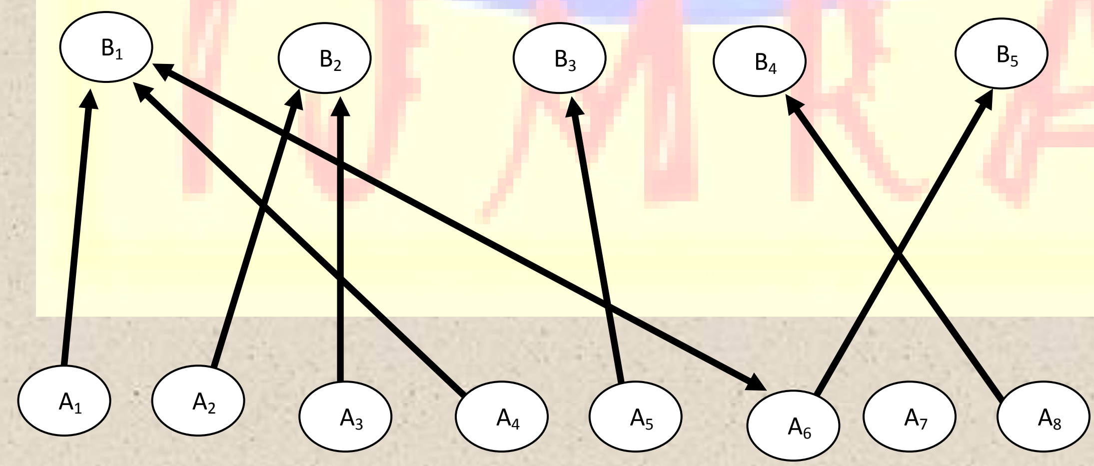

Fig 2: Employee -employer relationship model representation by a directed graph

Fig2 represents a directed graph of employee-employer Relational model. Nodes A_1, A_2, A_3,…, A_8 represent the domain space that denotes the employee and B_1, B_2, B_3,…, B_8 denote the range space that represent the employer. Considering associated relational matrix R_1 of this directed graph that represents the relationship model of FRMs.

R_1 =

	B_1	B_2	B_3	B_4	B_5
A_1	1	0	0	0	0
A_2	0	1	0	0	0
A_3	0	1	0	0	0
A_4	1	0	0	0	0
A_5	0	0	1	0	0
A_6	0	0	0	0	1
A_7	1	0	0	0	0
A_8	0	0	0	0	0

TABLE 2: Associated relational matrix of FRMs

Here we have taken one case on this model taking some nodes are on or off and try to find out some hidden pattern.

Case 1:

Consider the node A_1 i.e. pay with allowances and bonus to the employee to be in the on state and rest of the nodes in the off state. Now A_1= (1 0 0 0 0 0 0 0), passing A_1 into the matrix R_1 we get: -

$$A_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$$

$$A_1R_1 = (1\ 0\ 0\ 0\ 0) = H_1$$

$$H_1R_1^T = (1\ 0\ 0\ 0\ 0)\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$= (1\ 0\ 0\ 1\ 0\ 0\ 1\ 0) = A_2$$

$$A_2R_1 = (1\ 0\ 0\ 0\ 0) = H_1$$

$A_1 \rightarrow H_1 \rightarrow A_2 \rightarrow H_1 \in A_1$ Here A_1 is a unique state vector of the equilibrium state of this system. Thus A_1 is the fixed point. We have seen that if pay with allowances and bonus to the employee the company gets maximum profit, thus the best performances and employee works more numbers of hours.

Similarly we have to consider the other cases to find the hidden pattern of the system.

6. A case study:

At present India needs huge quantity of steel for the construction of various fields. So maximizing allied steel production in each and every factory is essential which is directly dependent on the relationship between the employer and the employees. Here we are trying to give a form of best relationship between the employees and employers. For this study the raw data is obtained from the **DURGAPUR CUSTING & ALLIED industry** for the years 2000-2010, which converted into a relationship map. The relationship map takes edge values as positive real numbers. If the number of concepts in the domain space is M and that of the range space is N we get an M×N causal relational matrix. After obtaining the relational matrix, we find the average matrix for simplification of calculation. The average matrix is then converted into a fuzzy matrix. Finally using the different parameter α (membership grade) we identify the best form of relationship between employee and employer in small scale industry. N_1 ,…, N_{10} are the ten broad heads which are associated with the employer and M_1, … , M_8 which represents the attributes of the employees.

6.1 Solution Methodology:

M_1	Salaries and wage
M_2	Salaries with wages and bonus to the employee
M_3	Bonus to the employee

M_4	Provident Fund(PF) to the Employee
M_5	Employee welfare medical
M_6	Employee welfare LTA (Leave travel allowances)
M_7	Employee welfare others
M_8	Staff training expenses

The allied steel industry is having 8 types of attributes related to the employee.

Now there are 10 effects on the employer when he proposes to pay and get work by the employee to maximize the production level.

N_1	2000-2001 Production level is 8,56,35,985 tonnes
N_2	2001-2002 Production level is 8,61,42,103 tonnes
N_3	2002-2003 Production level is 8,52,94,312 tonnes
N_4	2003-2004 Production level is 8,82,36,102 tonnes
N_5	2004-2005 Production level is 8,80,36,102 tonnes
N_6	2005-2006 Production level is 9,13,24,487 tonnes
N_7	2006-2007 Production level is 9,34,21,506 tonnes
N_8	2007-2008 Production level is 8,13,77,592 tonnes
N_9	2008-2009 Production level is 8,75,94,311 tonnes
N_{10}	2009-2010 Production level is 8,01,67,939 tonnes

We obtain 8×10 relational matrix from the relational map using weights from the data and converting $M_1,\dots, M_8$ and $N_1,\dots, N_{10}$ into the relational map. For example, M_1 represents the salaries and wages which is 77.08%, M_3 represents the bonus to the employee which is 10.31% and so on. The relational matrix given as bellow:-

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
N_1	77.08	87.39	10.31	11.29	2.39	1.21	1.62	2.11
N_2	79.23	90.11	10.88	11.40	2.41	1.25	1.58	2.23
N_3	80.11	90.89	10.78	11.39	2.48	1.24	1.75	1.97
N_4	81.11	92.61	11.50	11.30	2.58	1.35	1.49	2.65
N_5	76.69	88.70	12.01	12.05	2.50	1.29	1.60	2.70
N_6	77.39	89.35	11.96	11.78	2.43	1.30	1.55	2.66
N_7	79.88	91.97	12.09	11.66	2.35	1.32	1.54	2.59
N_8	78.11	89.70	11.59	12.01	2.40	1.28	1.45	2.49
N_9	77.85	89.75	11.90	12.03	2.49	1.25	1.66	2.45
N_{10}	79.26	90.65	11.39	11.93	2.53	1.23	1.53	2.40

TABLE 3: The relational matrix of employee-employer relational matrix

We convert the relational matrix into the average matrix which is given as $\mathbf{X} = [x_{ij}]$.

X=

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
N_1	38.54	43.70	5.16	5.65	1.20	0.61	0.81	1.06
N_2	39.62	45.06	5.44	5.70	1.21	0.63	0.79	1.12
N_3	40.06	45.45	5.39	5.70	1.24	0.62	0.88	0.99
N_4	40.56	46.31	5.75	5.65	1.29	0.68	0.75	1.33
N_5	38.35	44.35	6.01	6.03	1.25	0.65	0.80	1.35
N_6	38.70	44.68	5.98	5.89	1.22	0.65	0.78	1.33

N_7	39.94	45.99	6.05	5.83	1.18	0.66	0.77	1.30
N_8	39.06	44.85	5.80	6.01	1.20	0.64	0.73	1.25
N_9	38.93	44.88	5.95	6.02	1.25	0.63	0.83	1.23
N_{10}	39.63	45.33	5.70	5.97	1.27	0.62	0.77	1.20

TABLE 4: Average matrix corresponding to the employee –employer relational matrix give in Table 3

Now we use mean ($\boldsymbol{\mu_j}$), standard deviation ($\boldsymbol{\sigma_j}$) of each of the columns of the M×N relational matrix obtained from TABLE 4 and parameter α (membership grade $\alpha \in [0,1]$) to convert the average matrix X into the fuzzy matrix (y_{ij}); where i represents the i^{th} row and j represent the j^{th} column. For different values of the parameter α where $\alpha \in [0, 1]$ we determine the values of the entry y_{ij} in the average matrix using the following rule:

$$y_{ij}=\begin{cases} 0 & if\ X \le \mu_j - \alpha * \sigma_j \\ \dfrac{X-(\mu_j - \alpha*\sigma_j)}{(\mu_j+\alpha*\sigma_j)-(\mu_j - \alpha*\sigma_j)} & if\ X \epsilon (\mu_j - \alpha * \sigma_j, \mu_j + \alpha * \sigma_j) \\ 1 & if\ X \ge \mu_j + \alpha * \sigma_j \end{cases} \quad \textbf{(1)}$$

μ_j is the average of each column. Thus for different values of α, we obtain different fuzzy matrices. Finally we add up the rows of each fuzzy matrix and we define the new fuzzy membership function and allocate a value between [0, 1] to each row sum. Here the highest membership grade gives the best form of relationship between employee and employer, which maximizes the production level.

First we calculate the μ corresponding to each column of the matrix. From TABLE 4 μ of the first column is

μ_1= (38.54+39.62+40.06+40.56+38.35+38.70+39.94+39.06+38.93+39.63)/10 = 39.339. In the similar way we have to find the mean of the other columns. μ_2= 45.06, μ_3=5.723, μ_4=5.845, μ_5=1.231, μ_6=0.639, μ_7=0.791, μ_8=1.216. Now σ_j is calculated as follows: -

σ_1 of column 1 when μ_1 = 39.339

$d = \mu_1 - x_{ij}$	d^2
0.799	0.638401
0.281	0.078961
0.721	0.519841
1.221	1.490841
0.989	0.978121
0.639	0.408321
0.601	0.361201
0.279	0.077841
0.409	0.167281
0.291	0.084681
$\sum d = 6.23$	$\sum d^2 = 4.805490$

TABLE 5

$$\sigma_1 = \sqrt{0.480549 - 0.388129}$$
$$= 0.304007$$

Calculating the other values of σ_j for the other values of μ_j:-

$\sigma_1 =$ 0.304007 when $\mu_1 = 39.339$ $\sigma_5 =$ 0.015748 when $\mu_5 = 1.231$

σ_2\= 0.447299 when $\mu_2 = 45.06$ $\sigma_6 =$ 0.010954 when $\mu_6 = 0.639$

$\sigma_3 =$ 0.156234 when $\mu_3 = 5.723$ $\sigma_7 =$ 0.025612 when $\mu_7 = 0.791$

$\sigma_4 =$ 0.059025 when $\mu_4 = 5.845$ $\sigma_8 =$ 0.062952 when $\mu_8 = 1.21$

Case: 1

We consider some cases of $\mu_j - \alpha * \sigma_j$ and $\mu_j + \alpha * \sigma_j$ When α = 0.1, 0.2, 0.35, 0.45, 0.5, 0.6, 0.7, 0.85, 0.9, 1

j	$\mu_j - \alpha * \sigma_j$	$\mu_j + \alpha * \sigma_j$
1	39.308599	39.369401

2	45.015270	45.104730
3	5.707377	5.738623
4	5.839028	5.850903
5	1.229425	1.232575
6	0.637905	0.640095
7	0.788439	0.793561
8	1.209705	1.222295

TABLE 6: Membership grade α = 0.1

y_{ij} =

	M_1	**M_2**	**M_3**	**M_4**	**M_5**	**M_6**	**M_7**	**M_8**
N_1	**0**	**0**	**0**	**0**	**0**	**0**	**1**	**0**
N_2	**1**	**0.5**	**0**	**0**	**0**	**0**	**0.304764**	**0**
N_3	**1**	**1**	**0**	**0**	**1**	**0**	**1**	**0**
N_4	**1**	**1**	**1**	**0**	**1**	**1**	**0**	**1**
N_5	**0**	**0**	**1**	**1**	**1**	**1**	**1**	**1**
N_6	**0**	**0**	**1**	**1**	**0**	**1**	**0**	**1**
N_7	**1**	**1**	**1**	**0**	**0**	**1**	**0**	**1**
N_8	**0**	**0**	**1**	**1**	**0**	**0.956621**	**0**	**1**
N_9	**0**	**0**	**1**	**1**	**1**	**0**	**1**	**1**
N_{10}	**1**	**1**	**0**	**1**	**1**	**0**	**0**	**0**

From the TABLE 7, the corresponding fuzzy relational matrix for the parameter value α = 0.1 is as follows:-

The corresponding row sum of the above matrix are r1 = 1, r2 = 1.804764, r3= 4, r4 = 6, r5= 6, r6= 4, r7= 5, r8= 3.95621, r9= 5, r10= 4.

Now we define the new fuzzy membership function for graphical illustration, which convert the row sum to take values in the interval [0, 1].

New fuzzy membership function for TABLE 8 as follows:-

$$\mu_x(r_i) = \begin{cases} \mathbf{1} & \\ \dfrac{r_i - \text{Rowsum of min value}}{\{\text{Rowsum of max value} - \text{Rowsum of min value}\}} & \text{if row sum of max or min value} \le r_i \le \text{row sum of max or min value} \\ \mathbf{0} & \end{cases} \quad \textbf{(2)}$$

Using this membership function the corresponding values of $\mu_x(r_i)$ = 0, 0.160953, 0.6, 1, 1, 0.6, 0.8, 0.591324, 0.8, 0.6 where i = 1,2,…,10 and with the help of the MATLAB we draw the graph for $\alpha = 0.1$ (see in Fig:3), where $\mu_x(r_i)$ represent the y axis and r_i represent the x axis.

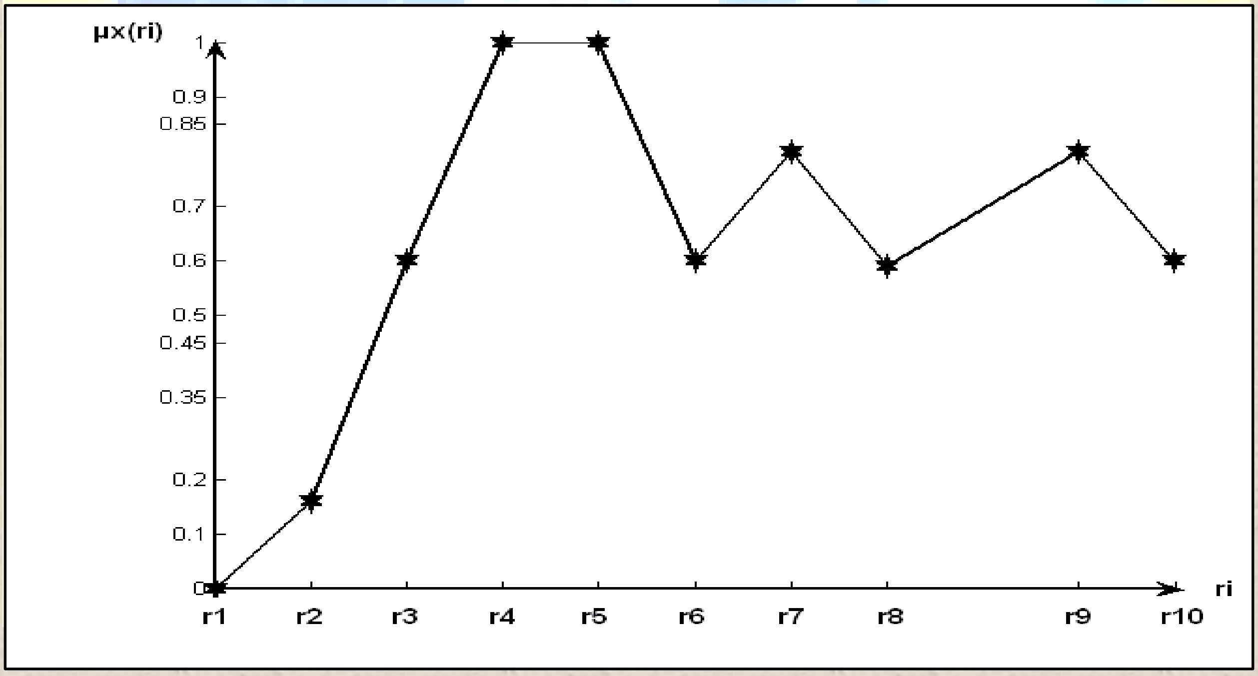

Fig 3: Graphical representation for α = 0.1

Result

From the graph of Fig.3 we see that **r4** and **r5** are the maximum i.e. the membership grade 1.and the lowest membership grade i.e. the membership grade 0 is the first row sum **r1**. In the similar way we have to check other fuzzy relational matrix for α=0.2, 0.35… and so on.

Case: 2

We have to calculate combined fuzzy matrix for all the values of α ∈ [0, 1] to find the best relationship.

y_{ij}=

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8
N_1	0	0	0	0	0	0	9.719430	0
N_2	9.962160	4.999999	0	0	0	0.149456	4.444333	0
N_3	10	9.920339	0	0	9.323869	0	10	0
N_4	10	10	7.095293	0	10	10	0	10
N_5	0	0	10	10	10	10	8.363317	10
N_6	0	0.103527	10	9.0753205	0.352899	10	1.222613	10
N_7	10	10	10	2.289411	0	10	0.152209	10
N_8	0.041129	1.032195	9.065737	10	0	6.299316	0	9.223668
N_9	0	1.366182	10	10	10	0.149456	10	7.496947
N_{10}	9.978607	9.423391	3.141046	10	10	0	0.152209	2.289201

TABLE 8: Fuzzy relational matrix for α∈ [0, 1]

The corresponding row sum of the above matrix are r1 = 9.719430, r2 = 19.555948, r3 = 39.2444208, r4 = 57.095293, r5 = 58.363317, r6 = 41.432244, r7 = 52.441620, r8 = 35.662045 r9 = 49.012585, r10 = 44.984454 Using the membership function the corresponding values of $\mu_x(r_i)$ is:- 0, 0.202215, 0.606958, 0.973933, 1, 0.651938, 0.878264, 0.533317, 0.807772, 0.724963 ,where i = 1, 2, …, 10.

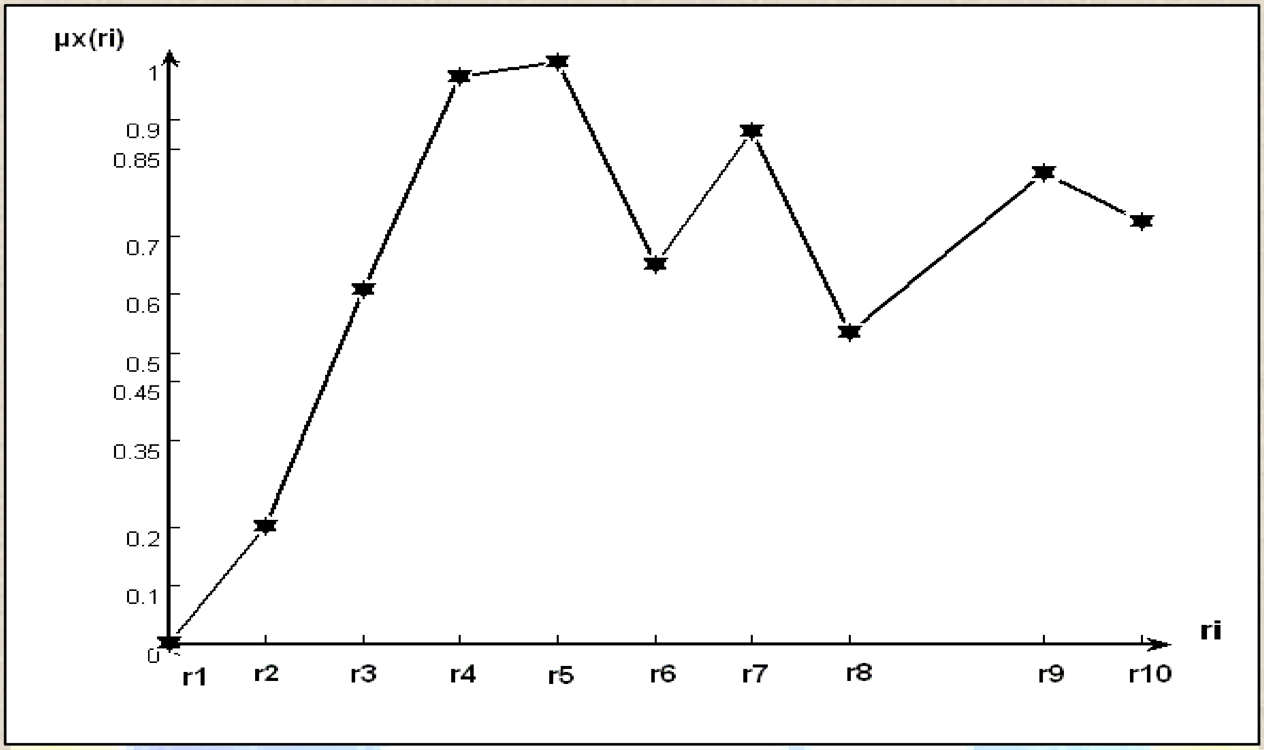

Fig 5: Graphical illustration for the values of α∈ [0, 1]

Result

Here we have to find the same result comparing the previous result i.e. fifth row sum r5 is getting the highest membership grade i.e. the membership grade is 1, and the first row sum r1 is the lowest membership grade i.e. the membership grade is 0.

6.2 Conclusions:

The highest membership grade gives the best form of relationship between employers and the employees, which maximizes the production level of the industry. Using the different parameter α = 0.1, 0.2, 0.35, …, 1 and from the combined fuzzy matrix, we observe that the row sum r5 gets the highest membership grade that is 1, which is in the year 2004-2005 giving the maximum production with maximum satisfaction of employees. The row sum r1 gets the lowest membership grade zero the year 2000-2001, the employee's satisfaction was poor with minimum production.

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